

# Multiple photoexcitation of two-dimensional electron systems: bichromatic magnetoresistance oscillations revisited

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We analyze theoretically magnetoresistance of high mobility two-dimensional electron systems being illuminated by multiple radiation sources. In particular, we study the influence on the striking effect of microwave-induced resistance oscillations. We consider moderate radiation intensities without reaching the zero resistance states regime. We use the model of radiation-driven Larmor orbits extended to several light sources. First, we study the case of two different radiations polarized in the same direction with different or equal frequencies. For both cases we find a regime of superposition or interference of harmonic motions. When the frequencies are different, we obtain a modulated magnetoresistance response with pulses and beats. On the other hand, when the frequencies are the same, we find that the final result will depend on the phase difference between both radiation fields going from an enhanced response to a total collapse of oscillations, reaching an outcome similar to darkness. Finally, we consider a multiple photoexcitation case (three different frequencies) where we propose the two-dimensional electron system as a potential nanoantenna device for microwaves.

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## I. INTRODUCTION

Photo-excited transport in two-dimensional electron system (2DES) is currently a fundamental topic from experimental and theoretical perspectives[1]. This interest comes not only from the basic explanation of a physical effect but also from the potential application for the development of future technological devices. When a Hall bar (a 2DES with a uniform and perpendicular magnetic field ( $B$ )) is irradiated with microwaves, different effects can be observed. Among them special attention deserves the recently discovered, microwave-induced (MW) resistance oscillations (MIRO) and zero resistance states (ZRS) [2–4]. Those effects, that show up at low  $B$  in high mobility samples, caused a big impact in the condensed matter community, mainly because they were obtained without quantization in the Hall resistance. Different theories have been proposed to explain the origin and physical consequences of these striking effects [5–12] but the physical origin is still being questioned and, in spite of the progress, there remain many aspects that could be better understood.

Thus, to unveil the physics behind them, a great effort has been made specially from the experimental side adding new features and different probes to the basic experimental setup[13–22]. The obtained experimental results are real challenges for the available theories. Thus, as an example, it has been recently published experimental results on the dependence of the oscillations with radiation power[23] where a very solid result has been obtained in terms of a sublinear relation, similar to a square root. This result has been also recently confirmed by other experiments on two-dimensional bilayer systems[24]. Yet, some theories predicted a linear

dependence[26] between MIRO and radiation power.

One of the most interesting setups that has been carried out, consists in illuminating the sample with two different light sources[27]. Some of the experimental results were realized at high radiation intensities making the MW-response to evolve into the zero resistance states regime[28]. The unexpected obtained results consisted in a transformed magnetoresistance ( $R_{xx}$ ) profile with new features including different peak positions and intensities and new zero resistance state regions. At first they were explained in terms of a theory of current domains formation[12, 29]. Other theories offered an alternative explanation based in the superposition of two radiation-driven motions acting on the center of the Larmor orbits[30]. Yet, a regime of moderate radiation intensities without reaching zero resistance states has not been yet sufficiently studied in experiments. We think that, if such an experiment were carried out, their results, about how the resistance profile is transformed, would shed some light on the influence of adding extra light sources on the magnetoresistance and, in the end, in the origin of MIRO. Of course the obtained experimental results would mean a real challenge for the existent theoretical models. Thus, a comparison of experiment with theory could help to identify the importance of the invoked-mechanisms in these theories. On the other hand, theories not only have to reasonably explain some experimental results. A solid theoretical model has to offer also definite predictions to be obtained if some parameters or variables are changed in a certain experimental set up. These predictions serve as orientation to experimentalists. And, if the theoretical predictions are confirmed, they also help to identify and interpret which mechanism is behind some definite physical effect.

This is the main goal of the present article, where we offer some theoretical predictions about the changes to be produced in MIRO when a 2DES is subjected to multiple radiations. Thus, in this article, we theoretically study magnetoresistance of a Hall bar being illuminated simultaneously by several radiation sources. We apply the model developed by the authors, the MW-driven Larmor orbits[5, 30], which is extended to a multiexcitation situation with moderate radiation intensities. According to this theory[5, 30] when a Hall bar is illuminated, the corresponding orbit centers of the Landau states (electronic oscillators) perform a classical trajectory consisting in a harmonic motion in the direction of the current. Thus, the whole 2DES moves periodically at the MW frequency. Our work is just focussed on how this response is going to be altered by the presence of extra radiation sources and eventually how this will be reflected in the magnetoresistance oscillations. We predict that in the presence of several radiation fields, the final motion of the center of the Larmor orbits will consist in the superposition of several harmonic motions, giving rise to interference effects. Accordingly, different response will be obtained depending on the relative frequencies, phase difference, radiation intensities, etc. We begin by considering only two sources of radiation that can have different or equal frequencies. In the first case we obtain a modulated  $R_{xx}$  response with pulses and beats. In the second case, the phase difference between the two light sources plays a key role. Thus, depending on its value we can achieve from an enhanced response (constructive interference) to a total collapse of the oscillations (destructive interference) with a result similar to darkness. This is a striking result that has not been obtained yet in experiments. Finally, we consider a multiple photoexcitation (three light sources) case with different frequencies where we propose the two-dimensional electron system as a multifrequency radiation sensor or nanoantenna device for the microwave range of radiation.

A potential experiment with phase difference could be envisaged as follows. The idea would be to start with one microwave source. The beam would be split into two parts using a "splitter", one that splits an input signal into two equal phase output signals. Then insert a phase shifter in the path of both beams. The reason to put it in the path of both beams rather than just one is because the phase shifter, even when it is not shifting phase, will introduce some power loss, and we want matched power in both beams. The phase shifter can be used to introduce an arbitrary phase ( $0^\circ < \theta < 180^\circ$ ) shift in either beam by turning a knob. Next, we can take the two beams separately to the sample and illuminate the sample with them.

Microwave-driven Larmor orbits model describe MIRO as a borderline effect between Classical Mechanics and Quantum Mechanics. Thus, this model assigns an essential classical feature to MIRO, such as the classical

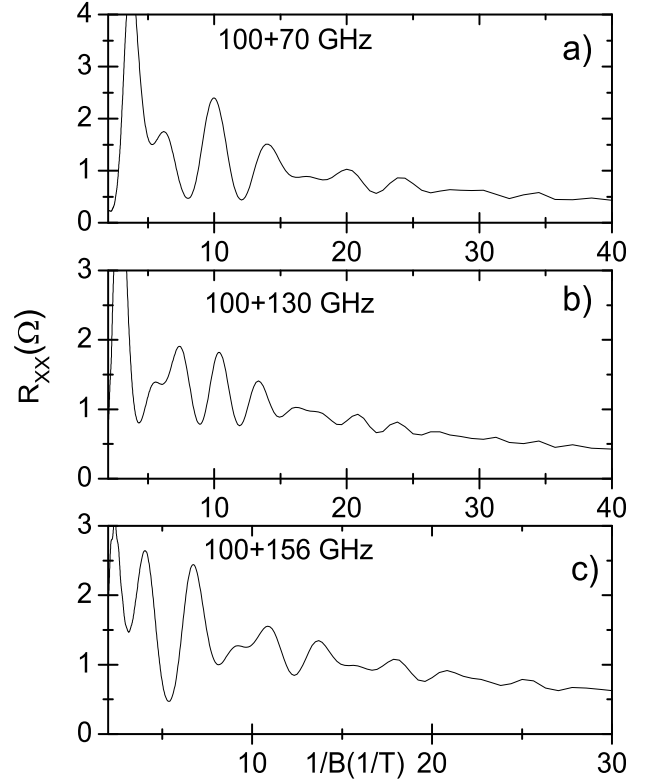


FIG. 1: Calculated longitudinal magnetoresistance versus the inverse of the magnetic field for two light sources having different frequencies. These frequencies, from a) to c) panels, are: 100 + 70 GHz, 100 + 130 GHz and 100 + 156 GHz. We have used large MW frequencies to have a sufficient number of oscillations to observe oscillations beats. The peak separation is going to be the same making beats more clearly seen in the three panels, although the peak intensity is smeared out for higher values of  $1/B$ .

trajectory that the electron orbits center guide performs driven by radiation. The rest of the model is basically based in Quantum Mechanics. The other models presented in the literature can be classified in either "displacement models"[6] or "inelastic models"[26]. These two theories are fully based in Quantum Mechanics. The results predicted in this paper are directly connected to the classical trajectory of the electronic orbits. Thus, these expected effects, that depend on equal or different frequencies and in the relative values of phase differences, are totally classical effects. Therefore if predictions are correct, neither the displacement model nor the inelastic mechanism would be able to fully explain the physics beneath them.

## II. THEORETICAL MODEL

The *MW driven Larmor orbits model*, was developed to explain the  $R_{xx}$  response of an irradiated 2DEG at low  $B$ . We obtained the exact solution of the corresponding electronic wave function when the 2DES is being illuminated by radiation[5, 30–33]:

$$\Psi_N(x, t) \propto \phi_n(x - X - x_{cl}(t), t) \quad (1)$$

, where  $\phi_n$  is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator,  $X$  is the center of the orbit for the electron motion.  $x_{cl}(t)$  is the classical solution of a forced harmonic oscillator[5, 30, 32, 33]. When one considers that the system is being driven by two different time dependent forces (two light sources)  $x_{cl}(t)$  has now the expression:

$$\begin{aligned} x_{cl}(t) &= x_1(t) + x_2(t) = \\ &= \frac{eE_1}{m^* \sqrt{(w_c^2 - w_1^2)^2 + \gamma^4}} \cos w_1 t \\ &+ \frac{eE_2}{m^* \sqrt{(w_c^2 - w_2^2)^2 + \gamma^4}} \cos w_2 t = \quad (2) \\ &= A_1 \cos w_1 t + A_2 \cos w_2 t \quad (3) \end{aligned}$$

where  $e$  is the electron charge,  $\gamma$  is a phenomenologically-introduced damping factor for the electronic interaction with the lattice ions emitting acoustic phonons and  $w_c$  is the cyclotron frequency.  $E_1$  and  $E_2$  are the electric fields amplitudes of the two radiation sources. Then, the obtained wave function is the same as the standard harmonic oscillator where the center is displaced by  $x_{cl}(t)$ . Thus, the orbit centers are not fixed, but they oscillate harmonically at the radiation field frequency  $w$ , if we only had one radiation field. In the case of two radiation fields, the classical trajectory of the orbit center would be the result of the superposition of two harmonic motions producing different types of interference effects that will be reflected in the final  $R_{xx}$  response.

This *radiation – driven* behavior will affect dramatically the charged impurity scattering and eventually the conductivity. Therefore, we introduce the scattering suffered by the electrons due to charged impurities randomly distributed in the sample. If the scattering is weak, we can apply time dependent first order perturbation theory. Thus, first we calculate the impurity scattering rate  $W_{N,M}$  between two *oscillating* Landau states  $\Psi_N$ , and  $\Psi_M$ [5, 30, 34–36]. Next we find the average effective distance advanced by the electron in every scattering jump that in the case of two MW sources is given by:

$$\Delta X^{MW} = \Delta X^0 + A_1 \cos w_1 \tau + A_2 \cos w_2 \tau \quad (4)$$

, where  $\Delta X^0$  is the effective distance advanced when there is no MW field present and  $\tau = 1/W_{N,M}$  is the scattering time. Finally the longitudinal conductivity  $\sigma_{xx}$  is

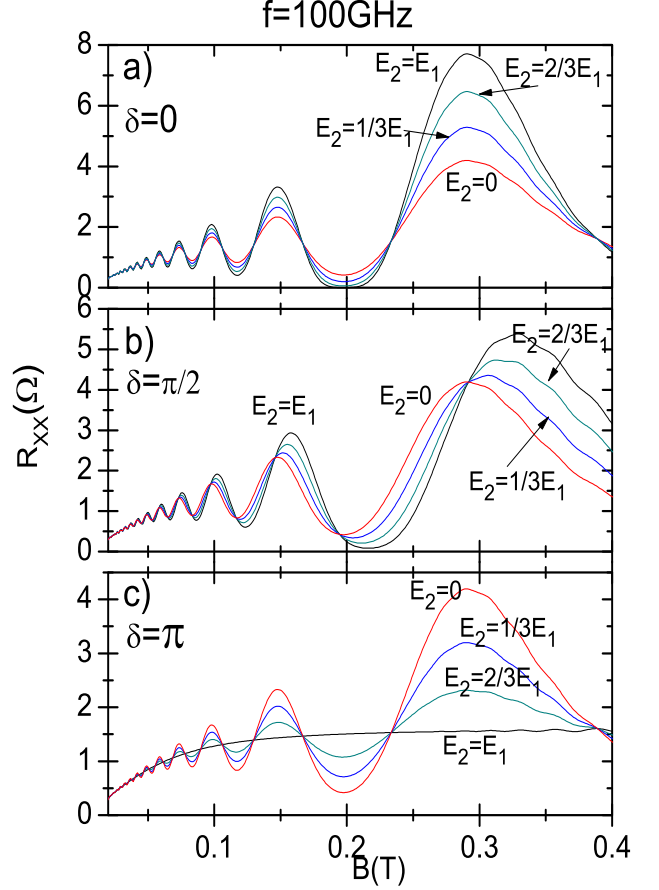


FIG. 2: Calculated  $R_{xx}$  versus magnetic field for two different light sources illuminating the sample. Fig. 2a, the phase difference  $\delta = 0$ , the interference is constructive giving an enhanced  $R_{xx}$  response.  $E_1$  and  $E_2$  represent the corresponding radiation electric fields. From  $E_2 = 0$  to  $E_2 = E_1$ , MIRO amplitudes increase, reaching ZRS around  $B = 0.2T$ . Fig. 2b,  $\delta = \pi/2$ . As  $E_2$  increases, the amplitude of, MIRO also increases and for  $E_2 = E_1$ , ZRS are almost reached at about  $B = 0.22T$ . On the other hand, there is also an increasing shift to the right of the whole MIRO response coming from the phase difference which shows up in equation (14). Fig. 2c,  $\delta = \pi$ , the interference is destructive and as the intensity  $E_2$  increases, MIRO are progressively smaller till the oscillations totally collapse.

given by:  $\sigma_{xx} \propto \int dE \frac{\Delta X^{MW}}{\tau}$  being  $E$  the energy. To obtain  $R_{xx}$  we use the relation  $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$ , where  $\sigma_{xy} \simeq \frac{n_i e}{B}$  and  $\sigma_{xx} \ll \sigma_{xy}$ .

Now we can proceed to consider some interesting cases depending on the relative frequencies of the light sources, phase difference, intensities etc. In all cases we are going to find that the center of the electronic orbits is subject to

more than one harmonic time-dependent force, each trying to move the center in its own direction, giving rise to interference effects. Eventually this will be observed in the  $R_{xx}$  oscillations producing different responses depending on the relative values (frequency, phase difference, intensity) of the radiation fields. Thus, if  $w_1$  is not very different from  $w_2$  and the MW fields intensities are equal, then we can write,  $A_1 \simeq A_2 = A$  and therefore:

$$\begin{aligned} x_{cl}(t) &= A[\cos w_1 t + \cos w_2 t] \\ &= 2A \cos \left[ \frac{1}{2}(w_1 - w_2)t \right] \cos \left[ \frac{1}{2}(w_1 + w_2)t \right] \end{aligned} \quad (5)$$

showing that now the oscillatory movement for the Larmor orbits center presents modulated amplitude with a frequency given by  $\frac{1}{2}(w_1 - w_2)$  whereas the main oscillation goes like  $\frac{1}{2}(w_1 + w_2)$ . This result is reflected in the average advanced distance by the electron in each scattering jump:

$$\Delta X^{MW} = \Delta X^0 + 2A \cos \left[ \frac{1}{2}(w_1 - w_2)\tau \right] \cos \left[ \frac{1}{2}(w_1 + w_2)\tau \right] \quad (6)$$

Thus, in this case we will observe  $R_{xx}$  oscillations with pulses and beats.

Another interesting regime takes place when the two radiation fields have the same frequency:  $w_1 = w_2$ . As in the latter case we have an interference effect which now depends on the phase difference ( $\delta$ ) between the two light sources and that can be tuned to obtain opposite effects in the  $R_{xx}$  response. The phase difference between the radiation fields is translated into the expression of  $x_{cl}$  writing now:

$$x_{cl}(t) = A_1 \cos wt + A_2 \cos(wt + \delta) \quad (7)$$

If  $\delta = 0 \Rightarrow x_{cl}(t) = (A_1 + A_2) \cos wt$  and the interference is constructive. Then, the average distance advanced in every scattering event in the direction of the current will be

$$\Delta X^{MW} = \Delta X^0 + (A_1 + A_2) \cos w\tau \quad (8)$$

Therefore, we will obtain an enhanced response in  $R_{xx}$  regarding the case of just one light source with the same frequency and intensity.

If  $\delta = \pi \Rightarrow x_{cl}(t) = (A_1 - A_2) \cos wt$  and the interference is destructive. Then,

$$\Delta X^{MW} = \Delta X^0 + (A_1 - A_2) \cos w\tau \quad (9)$$

Therefore, we will obtain a reduced response in  $R_{xx}$  regarding the case of just one light source with the same frequency and intensity. In the particular case of equal intensities,  $A_1 \simeq A_2$ , we will obtain a striking result in the nearly total destruction of MIRO and the response

will be the same as in darkness,  $\Delta X^{MW} = \Delta X^0$ .

For an arbitrary value of the phase difference is straightforward to obtain a simpler expression for  $x_{cl}$  with new amplitude  $A$  and phase difference  $\alpha$ :

$$x_{cl}(t) = A \cos(wt + \alpha) \quad (10)$$

where,

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta} \quad (11)$$

and,

$$\tan \alpha = \frac{A_2 \sin \delta}{A_1 + A_2 \cos \delta} \quad (12)$$

Therefore, for  $\delta = \frac{\pi}{2}$ , we readily obtain,

$$x_{cl}(t) = \sqrt{A_1^2 + A_2^2} \cos \left( wt + \arctan \left[ \frac{A_2}{A_1} \right] \right) \quad (13)$$

$$\Delta X^{MW} = \sqrt{A_1^2 + A_2^2} \cos \left( w\tau + \arctan \left[ \frac{A_2}{A_1} \right] \right) \quad (14)$$

Then, we expect for this case not only a different MIRO amplitude but also an oscillations shift due to the presence of the phase difference  $\alpha$ .

The theory can be extended to a higher number of radiation sources to have a multiexcitation of the sample. Thus, we can write for  $n$  sources:

$$x_{cl}(t) = \sum_{i=1}^n A_i \cos w_i t \quad (15)$$

$$\Delta X^{MW} = \sum_{i=1}^n A_i \cos w_i \tau \quad (16)$$

In this case we will have a linear superposition of all radiations giving rise to a more pronounced interference effects.

## CALCULATED RESULTS

In Fig. 1 we present calculated longitudinal magnetoresistance versus the inverse of the magnetic field for two light sources having different frequencies. These frequencies, from a) to c) panels, are: 100 + 70 GHz, 100 + 130 GHz and 100 + 156 GHz. We clearly observe that the amplitudes are modulated in the three pairs of frequencies. We have used large MW frequencies to have a sufficient number of oscillations that permit us to observe oscillations beats. Versus  $(1/B)$  the peak separation is going to be the same making beats more clearly seen in the three panels, although the peak intensity is smeared out for higher values of  $1/B$ .



In Fig. 2 we present calculated  $R_{xx}$  versus magnetic field for two different light sources with the same frequency. We observe that depending on the phase difference ( $\delta$ ) between the two radiation fields different outcomes are obtained. Thus, in Fig. 2a,  $\delta = 0$ , the interference is constructive, (see eq. (9)), and we obtained an enhanced  $R_{xx}$  response. In the figure  $E_1$  and  $E_2$  represent the corresponding radiation electric fields. We represent curves from  $E_2 = 0$  to  $E_2 = E_1$ , observing that MIRO amplitudes increase and for equal electric field intensities, ZRS are reached, around  $B = 0.2T$ . In Fig. 2b,  $\delta = \pi/2$ , we represent the same cases as in 2b. We observe that as  $E_2$  increases two effects take place. First, the amplitude of MIRO increases, as expected from equation (14), and for  $E_2 = E_1$ , ZRS are almost reached around  $B = 0.22T$ . Second, there is also an increasing shift to the right of the whole  $R_{xx}$  response coming from the phase difference,  $\arctan\left[\frac{A_2}{A_1}\right]$ , which shows up in equation (14). Finally, in Fig. 2c,  $\delta = \pi$ , the interference is destructive (see eq. (9)) and, as the intensity  $E_2$  increases, MIRO are progressively smaller till the oscillations totally collapse. Thus, although the sample is illuminated, the obtained  $R_{xx}$  response corresponds to darkness.

As we said above, the theory can be extended to a higher number of radiation sources. If all radiations have different frequencies we can define it as *multichromatic excitation* of the sample:  $\Delta X^{MW} = \sum_{i=1}^n A_i \cos w_i \tau$ . In this case we will have a linear superposition of all radiations giving rise to more pronounced interference effects. As an illustrative example we can consider the case of three radiation waves with different frequencies. The most interesting case could be of an *amplitude modulated radio signal*. It is well known that the electric field of such a radiation can be mathematically represented by[37]:

$$E(t) = B_1 \cos w_1 t + \frac{1}{2} B_2 (\cos w_2 t + \cos w_3 t) \quad (17)$$

which is the sum of three harmonic contributions of different frequencies. Therefore, the modulated signal has three harmonic components, a carrier wave and two sinusoidal waves known as sidebands whose frequencies are slightly above and below of the carrier wave frequency. In this particular case these frequencies are related among them by definite relations:  $w_1 = w_c$  which is the frequency of the carrier wave, then  $w_c$  is known as *carrier wave frequency*. The other two frequencies corresponds to the sidebands:  $w_2 = w_c + w_m$  and  $w_3 = w_c - w_m$  where  $w_m$  is known as *message wave frequency*.  $B_1$  and  $B_2$  are the corresponding amplitudes.

According to the above results, we have shown that the electronic oscillators can be driven by more than one electromagnetic waves, absorbing the corresponding energy from them[38]. The theoretical model of *Radiation driven Larmor orbits* states that the electromagnetic radiation will translate electric field intensity and frequency into

the motion of the electronic orbits. In other words, *information* transported in the electromagnetic waves in form of amplitude of frequency can be transferred in an efficient way into the 2DES through the center guide of the electronic orbits. Therefore, we have demonstrated that a hall bar can perform efficiently as a multifrequency radiation sensor or nanoantenna in the MW range of radiation. Another important issue with interest not only from basic knowledge but also from a technological standpoint is the possibility of reemission for these systems. The topic of reemission has not been yet addressed by any theory. Yet, the microwave-driven Larmor orbit model can do it considering that the radiation-driven back and forth motion of the electronics orbits makes them oscillating dipoles. Importantly, the possibility of harmonic excitation and frequency multiplication with the corresponding reemission can be addressed by the present model. These features have been already predicted by this theory[39] when these systems reach slightly anharmonic regimes. All these topics constitute the core of a future work.

## CONCLUSIONS

In summary, We have studied  $R_{xx}$  of high mobility 2DES being illuminated by multiple radiation sources. We have essentially studied the influence on the microwave-induced resistance oscillations in a moderate intensity radiation regime, which excludes ZRS. We have applied the model of radiation-driven Larmor orbits extended to several radiations (multichromatic). First, we study the case of two different radiations with different or equal frequencies. For both cases we find a regime of interferences of harmonic motions acting on the center guide of the electronic orbits. When the frequencies are different, we obtain a modulated magnetoresistance response with pulses and beats. On the other hand, when the frequencies are the same, we find that the result will depend on the phase difference between both radiation fields going from an enhanced response to a total collapse. Finally, we consider a multiple photoexcitation case where we propose the two-dimensional electron system as a potential nanoantenna device for microwave radiation.

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